LETTER

On the Sparse Signal Recovery with Parallel Orthogonal Matching Pursuit*

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SUMMARY In this letter, *parallel orthogonal matching pursuit (POMP)* is proposed to supplement orthogonal matching pursuit (OMP) which has been widely used as a greedy algorithm for sparse signal recovery. Empirical simulations show that POMP outperforms the existing sparse signal recovery algorithms including OMP, compressive sampling matching pursuit (CoSaMP), and linear programming (LP) in terms of the exact recovery ratio (ERR) for the sparse pattern and the mean-squared error (MSE) between the estimated signal and the original signal.

key words: *compressed sensing, orthogonal matching pursuit, mean squared error, multiple candidates*

1. Introduction

Compressed sensing has been known as a revolutionary technique for efficiently acquiring and reconstructing a sparse signal by finding solutions of underdetermined linear systems [1], [2]. Compressed sensing basically consists of two parts: random projection by a linear measurement matrix and sparse signal recovery. Random projection can be expressed as a product of an n-dimensional sparse vector, $x \in \mathbb{R}^n$ having at most k non-zero elements ($k \ll n$) and an $m \times n$ matrix A ($m < n$). It is given as

$$y = Ax + z, \tag{1}$$

where $y \in \mathbb{R}^m$ indicates the observation vector and z denotes Gaussian noise [3]. Signal recovery in compressed sensing is used to obtain the original vector x from the observation vector y. While the signal recovery problem in compressed sensing can be solved with a convex optimization with linear programming (LP) technique, its computational complexity significantly increases according to size of the original vector (i.e., $O(n^3)$).

Recently, as alternatives to the LP-based recovery technique, several greedy algorithms have been proposed for reducing the complexity of sparse signal recovery [4]–[7]. Among them, orthogonal matching pursuit (OMP) has been known as a representative greedy algorithm owing to its simplicity and competitive performance. However, one of the most challenging issues associated with the OMP algorithm is to select the column of A that has the highest correlation with the current residual at each step and to eliminate its effect when generating the next residual. In particular, in the first iteration, there exists a considerable amount of interference in the residual (in fact, y), and the resultant estimate for the original vector x is too rough to eliminate its effect in the residual. It becomes more problematic in the presence of noise. In this letter, we propose an algorithm called parallel orthogonal matching pursuit (POMP) that selects multiple candidates among columns of A that have a high correlation with y at the first iteration. This reduces the uncertainty of the first iteration in the OMP algorithm.

2. Parallel Orthogonal Matching Pursuit (POMP)

In POMP, M columns from a sensing matrix A are selected in the first iteration, which have a high correlation value with the observation signal vector y. Then, conventional OMP processes are carried out in parallel, on the basis of the selected columns in the first iteration. Figure 1 shows an example of index set generation in POMP. a_i indicates the i-th column of A and Λ_t^j denotes the j-th candidate index set at the t-th iteration. OMP selects a_6, a_2, and a_5 in the first, second, and third iterations, respectively. The selection process is repeated k times for the k-sparse vector in OMP. On the other hand, POMP selects three columns (a_6, a_3, and a_8) simultaneously in the first iteration as shown in Fig. 1 when $M = 3$. In the second iteration, a_2, a_4, and a_7 are added to the first, second, and third candidate sets, respectively. Thus, M candicate sets are maintained in POMP.

It is possible that the same index set is generated during iteration in POMP. For example, if a_6, a_2 are added to the first index set in the first and second iterations, respectively and a_2, a_6 are added to the second index set in the first and second iteration, respectively, then the first index set Λ_2^1 becomes the same as the second index set Λ_2^2 after the second iteration. In POMP, however, if a candidate set has the same

Iteration	OMP	POMP (M=3)
1	$\Lambda_1 = \{a_6\}$	$\Lambda_1^1 = \{a_6\}, \Lambda_1^2 = \{a_3\}, \Lambda_1^3 = \{a_8\}$
2	$\Lambda_2 = \{a_6, a_2\}$	$\Lambda_2^1 = \{a_6, a_2\}, \Lambda_2^2 = \{a_3, a_4\}, \Lambda_2^3 = \{a_8, a_7\}$
3	$\Lambda_3 = \{a_6, a_2, a_5\}$	$\Lambda_3^1 = \{a_6, a_2, a_5\}, \Lambda_3^2 = \{a_3, a_4, a_{10}\}, \Lambda_3^3 = \{a_8, a_7, a_1\}$
⋮	⋮	⋮

Fig. 1 Example of index set generation of POMP in the case when $M = 3$.

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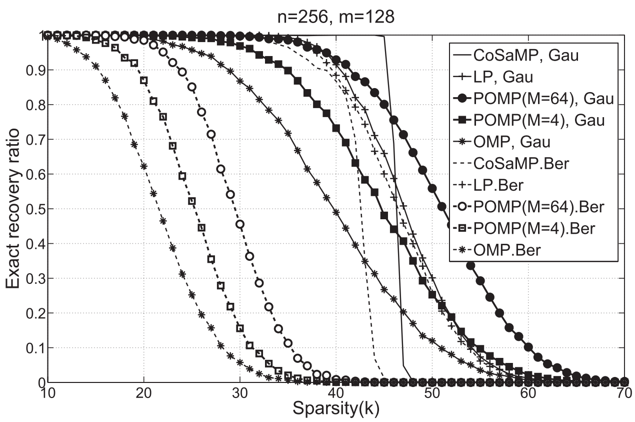

Fig. 2 Exact recovery ratio (ERR) of POMP for varying sparsity without noise.

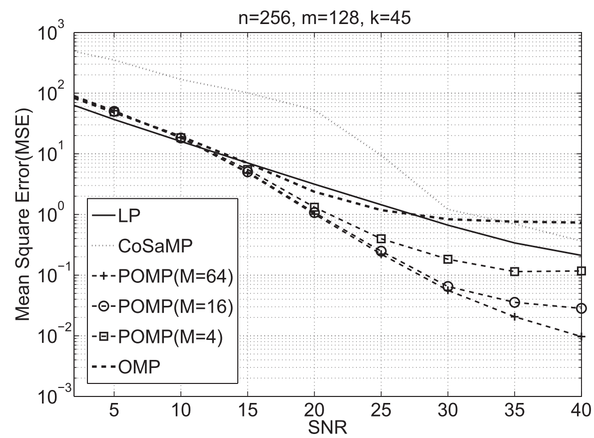

Fig. 3 Mean squared error (MSE) of POMP with noise for varying SNR values.

elements as those in other candidate sets in the l-th iteration, then the added element in the l-th iteration is removed and another element that has high correlation with the residual is added for maintaining *distinct M* candidate sets. After k iterations, the estimate for the original vector $\hat{x}$ is obtained by solving a least-square problem with the index set resulting in the minimum residual among the M candidate sets. Clearly, there exists a trade-off between the recovery performance and the computational complexity in POMP. POMP requires M times more complexity than OMP. Therefore, we need to adjust M in POMP according to the system requirements such as speed, storage requirement, and recovery performance.

3. Experimental Results

Empirical simulations have been extensively carried out to verify the performance of POMP. The conventional OMP, compressive sampling matching pursuit (CoSaMP), and LP-based l_1-minimization [8] were considered as references. In each trial, we generate an $m \times n$ ($m = 128$ and $n = 256$) sensing matrix A with elements drawn independently from the Gaussian distribution $N \sim (0, 1/m)$. In addition, we generate a k-sparse signal vector x whose support is randomly chosen and each nonzero element is drawn from a standard Gaussian distribution $N \sim (0, 1)$.

Figure 2 illustrates the exact recovery ratio (ERR) without noise and thus, in this case, $z = \mathbf{0}$ in (1). The ERR is defined as the probability of the exact recovery on the position of non-zero elements [4]. For performance comparison, in this figure, we also tested the Bernoulli sparse signals whose support is randomly chosen and each nonzero element is drawn from the Bernoulli distribution (+1 or −1). In each recovery algorithm in Fig. 2, we performed 5000 independent trials. Figure 2 shows that POMP yields better performance than OMP, and the ERR performance improves with an increase in M. Suprisingly, as for the detection of Gaussian sparse signals, POMP can acheive better performance than LP if M is sufficiently large, even though POMP operates with much lower complexity than LP.

We also consider the performance of POMP in the presence of noise. Figure 3 shows the mean squared error (MSE) performance of various sparse signal recovery algorithms including POMP for varying signal-to-noise (SNR) values, which is obtained by comparing the original signal vector x and the estimated signal vector $\hat{x}$. The MSE and SNR are defined as

$$\text{MSE} = \mathbf{E}\left[\|x - \hat{x}\|^2\right] \quad \text{and} \quad \text{SNR} = \frac{\mathbf{E}\left[\|Ax\|^2\right]}{\mathbf{E}\left[\|z\|^2\right]}.$$

$\mathbf{E}[\cdot]$ denotes the statistical expectation operator. Sparsity k is assumed to be 40 in Fig. 3. In the presence of noise, POMP outperforms OMP, CoSaMP, and LP, regardless of the SNR values. As expected, the MSE performance of POMP improves with an increase in M.

4. Conclusion

In this letter, the parallel orthogonal matching pursuit (POMP) algorithm is proposed for improving performance of sparse signal recovery. From the extensive simulations, it is shown that POMP outperforms the existing sparse signal recovery algorithms such as OMP, CoSaMP, and LP in terms of ERR and MSE especially in the case of Gaussian sparse signals.

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